Measurement of Eardrum Acoustic Impedance

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Abstract

In this paper we describe a system which we have developed to measure cat ear canal specific acoustic impedance Z_{sp}, magnitude and phase, as a function of frequency, for frequencies between 200 Hz and 33 kHz, and impedance magnitude between 4.0 to 4.0×10^5 [Rayles] (MKS). The object to be measured is placed at the end of a 3.5 mm diameter sound delivery tube. After a simple calibration procedure, which determines the Thevenin parameters for the acoustic source transducer, the impedance may be calculated from the pressure measured at the orifice of the delivery tube with the unknown load in place. This procedure allows for a fast but accurate measure of a specific acoustic impedance. The system has been tested by measuring the impedance of a long cavity and comparing this response to the exact solution of the linearized Navier Stokes equations (acoustic equations including viscosity and thermal conduction). We have used this system to measure the impedance of the normal cat tympanic membrane in more than 30 cats. Healthy animals were found to have a real input impedance of ρc between 0.3 to 20.0 kHz. When the scala vestibuli was drained, the real part of the impedance dropped to less than $\rho c/10$ for frequencies less than 3.0 kHz. Above 3 kHz, the impedance for the drained cochlea is best described by an open circuited transmission line.

1 Introduction

As in the case of electrical networks, impedance is an important characterization of an acoustical network. However, unlike the electrical case, no convenient commercial system is available for acoustical impedance measurement. Reasons for this lack include the unavailability of low distortion (0.005% distortion) acoustic sources, the unavailability of precisely calibrated acoustic impedances, and the complications introduced by the wave-like nature of sound (due to the relatively slow sound speed). Therefore the need exists for a fast, precise, automatic (computer) method of linear system identification. In this paper we shall describe a method of acoustical measurement which address all of the above problems. The impedance measurement technique used here is based on accurately estimating the Thevenin equivalent parameters for a sound source, namely the open circuit pressure and the source impedance, as functions of frequency. If one knows the Thevenin equivalent source parameters, then the impedance of any acoustic load may be calculated given the pressure at its input. This technique has been previously applied [Beranek (1949); Lynch (1974); Mawardi (1949); Tonndorf and Khanna (1967)]; however the implementation discussed here differs in several ways. First, we use an electret push-pull (Hunt, 1954) low distortion sound source that has a uniform frequency response up to 30 kHz. Second, this sound source is connected to a uniform diameter (3.5mm) tube through a matching acoustic resistor which is used to reduce reflections at the source end of the sound delivery tube, thereby minimizing standing waves in the delivery tube [Sokolich, G.W. (1977)]. As a result of the matching resistor, the Thevenin source impedance at the sound

delivery end is close to the characteristic impedance of the acoustic transmission line.
The transducer's open circuit pressure and source impedance are computed from four different pressure responses, (measured via a single microphone at the system's orifice) which result from its being terminated by four different acoustic loads. The precise procedure for doing this is believed to be both novel and considerably more accurate than previous published methods. Based on the assumptions made in the calibration procedure, the calibration is insensitive to temperature changes and positioning of the probe microphone. Also, the method does not require the use of a calibrated probe microphone, since the measurement method (as we shall show) is independent of the probe transfer function.
We have developed this system to measure the specific acoustic impedance looking into the cat ear canal. Our results differ from those of others in significant ways. First, we find that the eardrum (TM) is matched to the impedance of air over the frequency range from 300 Hz to 20 kHz. However, significant animal variability was observed. For those animals for which the TM was clear (transparent), which we took as a measure of a healthy middle ear, the measured impedance was uniform over frequency and closest to ρc. The impedance was usually measured within 5 mm of the TM, with the bulla and septum widely opened. A closed bulla and septum strongly modified the measured impedance at certain frequencies, such as at the bulla resonance frequency of 3 kHz.

2 Theoretical method

In Fig. 1 we show an equivalent circuit for our sound delivery system, loaded by an unknown impedance $Z_x(\omega)$. The output response pressure $P_x(\omega)$ across the load $Z_x(\omega)$ is measured through a probe tube having transfer function $H_p(\omega)$. From Fig.1

$$R_x(\omega) = H_p(\omega)P_x(\omega) \tag{1}$$

where $R_x(\omega)$ represents the response measured through the probe tube at radian frequency $\omega = 2\pi f$. Note that Z_x, R_x, H_p, and P_x are all complex functions of frequency representing the Fourier transforms of time functions $z(t)$, $r(t)$, $h(t)$, and $p(t)$. From Fig. 1, the input sound source P_s

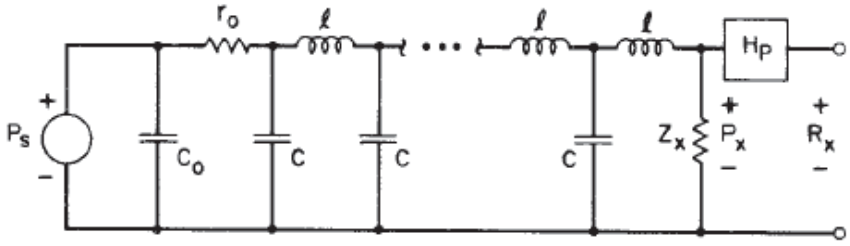

Figure 1: An equivalent circuit for our sound delivery system

is connected to the sound delivery tube, represented here as a mass-compliance transmission line, through a porous screen which acts as an acoustic resistor r_0. The screen is created by cascading several screens together which terminate the transmission line at its driven end. The proper screen resistor is determined by setting Z_x to an acoustic open circuit, (zero volume velocity via a rigid wall condition at the probe microphone) and then measuring the standing wave ratio (SWR) with the probe as a function of frequency. The screen is chosen to minimize the resulting SWR.
We define $Z_0(\omega)$ to represent the Thevenin complex source impedance and $R_0(\omega)$ to represent the Thevenin open circuit source pressure. For reasons which will become clear, we shall define all

impedances in terms of the *specific aoucsit impedance* [Beranek (1954)], which is defined as the ratio of the pressure to the particle velocity, and which is measured in MKS Rayles, or equivalently [Pa sec/m], where one Pa is the pressure in Pascals [1 N/m^2].
The relation between $R_x(\omega)$ and $Z_x(\omega)$ in terms of $R_0(\omega)$ and $Z_0(\omega)$ is,

$$R_x = \frac{Z_x R_0}{(Z_0 + Z_x)} \tag{2}$$

Given four known load impedances $Z_1(\omega)$ $Z_2(\omega)$ $Z_3(\omega)$, and $Z_4(\omega)$, which correspond to measured pressure responses R_1, R_2, R_3, and R_4, one may solve for $Z_0(\omega)$ and $R_0(\omega)$ in terms of the known R_1, R_2, R_3, R_4, Z_1, Z_2, Z_3, and Z_4 by solving the over specified system of equations:

$$\begin{bmatrix} Z_1 & -R_1 \\ Z_2 & -R_2 \\ Z_3 & -R_3 \\ Z_4 & -R_4 \end{bmatrix} \begin{bmatrix} R_0 \\ \\ Z_0 \end{bmatrix} = \begin{bmatrix} R_1\ Z_1 \\ R_2\ Z_2 \\ R_3\ Z_3 \\ R_4\ Z_4 \end{bmatrix} \tag{3}$$

by least squares methods, resulting in the solution:

$$\begin{bmatrix} R_0 \\ Z_0 \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} \Sigma|R_i|^2 - \Sigma Z_i^* R_i \\ \Sigma R_i^* Z_i - \Sigma|Z_i|^2 \end{bmatrix} \begin{bmatrix} \Sigma|Z_i|^2 R_i \\ \Sigma|R_i|^2 Z_i \end{bmatrix} \tag{4}$$

where

$$\Delta = (\Sigma|Z_i|^2)(\Sigma|R_i|^2) - (\Sigma R_i^* Z_i)(\Sigma Z_i^* R_i)$$

Once R_0 and and Z_0 have been determined, Z_x may be found from R_x using the relation

$$Y_x = Y_0(\frac{R_0}{R_x} - 1), \tag{5}$$

where Y_x and Y_0 are $\frac{1}{Z_x}$ and $\frac{1}{Z_0}$ respectively.
The above analysis assumes that we have four standard impedances Z_1, Z_2, Z_3, and Z_4 which are known. We next discuss the choice of these standard impedances.

3 Calibration impedances

For an acoustic transmission line closed at the far end, the specific input impedance is [Beranek, (1954)]

$$Z(\omega) = -i\ \rho c\ cot(kL). \tag{6}$$

In air, $\rho c = 412.5$ rayles, $i = \sqrt{-1}$, $k = \omega/c$ and L is the length of the tube. Since any transmission line model assumes uniform (plane-wave) flow, the specific impedance is independent of the cross-sectional area of the tube. Equation (6) is therefore a one parameter model of the impedance, where the length L may be approximately determined from the first antiresonance of the impedance (that frequency f_0 where Z first becomes zero). The length L is related to f_0 by the relation $L = c/4f_0$. Thus the specific acoustic impedance Eq. (6) is completely determined if we know the length, or equivalently the frequency f_0 of the first impedance zero. This frequency is easily estimated from the pressure response since the pressure has its zeroes at the impedance zeros.
These lengths may be very precisely determined by minimizing the norm over frequency of the residual error of the over determined equations (Eq. (3)), with respect to the unknown cavity lengths. The resulting estimated lengths then provide the best overall fit to the equations. This

procedure is used to improve the accuracy of the estimate of L_1, L_2, L_3, and L_4. It was found to be necessary to include damping in the model for Z_1, Z_2, Z_3, and Z_4. This was done by using the 'exact' solution to the transmission line equations with viscous and thermal conduction losses included rather than Eq. (6) [White et al., (1982); Zuercher et al., (1977)].
In Fig. 2 we show the open circuit pressure and source impedance magnitude as estimated by the above defined procedure for our transducer system. The units for pressure are in A/D volts, with 10 volts applied to the transducer, while the units for the source impedance are normalized by ρc, the impedance of air.

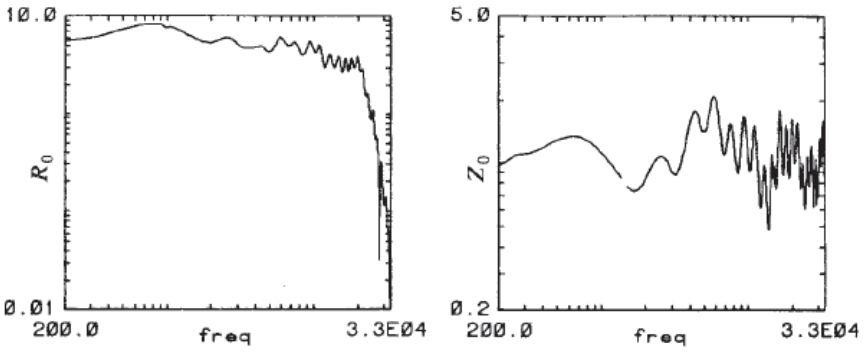

Figure 2: The open circuit pressure and source impedance magnitude

4 Cat impedance measurements

Next we present our experimental results. Periodically, prior to use, the system was re-calibrated to reduce the effects of temperature and system variations. The calibration procedure only took a few minutes and was not inconvenient. In order to estimate the accuracy of the calibration of the system, we initially measured the impedance of a 2.35 cm uniform cavity. Since the exact solution, including losses, is known for this case, it is possible to compare the measured impedance to the exact results. we show this comparison in Fig. 3. In this figure, the exact numerical solution is shown as a dashed line, and the experimental result is shown as the solid line. In the left panel, the magnitudes of the two impedances are shown, while in the right panel, the real part of the impedances are compared. The experimental and theoretical results are in very good agreement, with the two curves almost totally overlapping over three orders of magnitude.

We measured the impedance looking into a cat ear with the probe tube tip between 2.5 to 5.0 mm from the ear drum. In Fig. 4, curve 1, we see the measured impedance of a healthy cat ear over the frequency range from 200 Hz to 33 kHz. Over this frequency range, the input impedance is nearly real and is equal to ρc. The majority of healthy cat ears showed similar results, with damaged ears showing impedance frequency dependent variations of between 6 to 10 dB. While it is aesthetically pleasing to see such a uniform impedance match, it is also surprising.

The remaining curves of Fig. 4 show the results of a systematic damage experiment, where the cochlea was progressively removed from the system while the eardrum impedance was monitored. Curve 1 shows the normal eardrum impedance prior to damage. In curve 2, we show the measured impedance after touching the basilar membrane with a glass probe. In curve 3, the basilar membrane has been punctured with the glass probe, producing a small hole. Note that after touching the BM and after creating the hole, the eardrum has a small, but measurable change in the input impedance. Relative to the ear canal pressure, this difference was only 2 and 4 dB at 1.8 kHz,

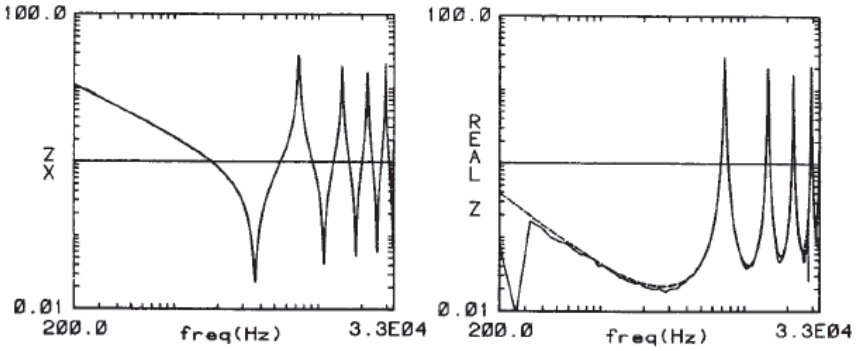

Figure 3: The experimental result

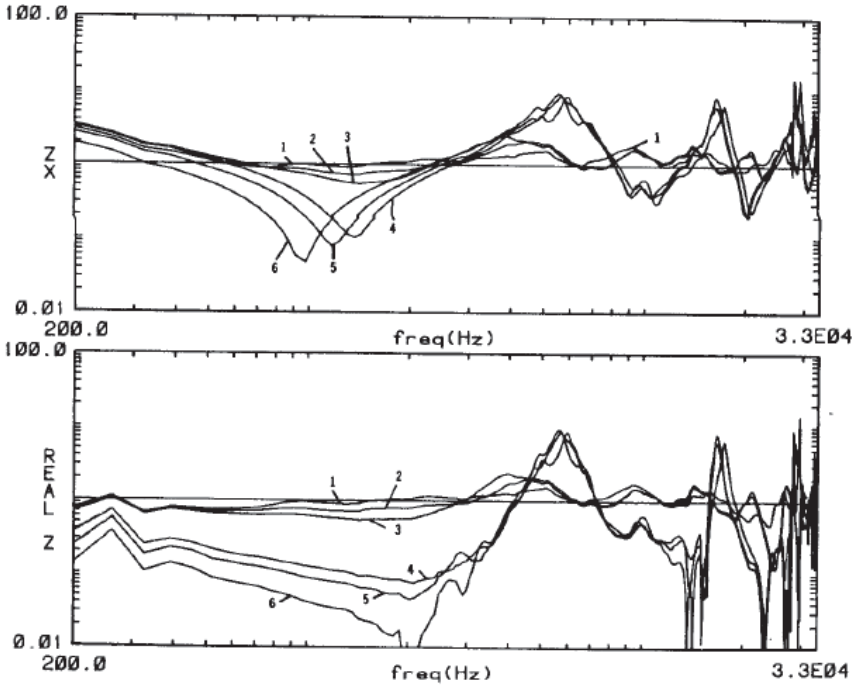

Figure 4: the impedance looking into a cat ear

while the impedance change was 3 and 6 dB respectively. After removal of the round window, no observable impedance change was observed. Next, the fluid was drained from the scala tympani. Again no observable change was seen. However, when the BM was removed, and the scala vestibuli was drained, a dramatic change in impedance was observed, as may be seen in curve 4. Below 3.5 kHz the resistance decreased by 20 dB. Above that frequency a standing wave pattern is observed in the magnitude response, which is similar to the cavity response seen in figure three, in that the resistance increases at the pole frequencies and decreases at the zero frequencies. From curve 4, it is clear that the largest component of the resistance is due to the cochlea.

In curve 5 we see the effect to cutting the Tensor Tympani. Its removal reduced the eardrum stiffness and slightly reduced the resistance. Finally cutting the stapes free (curve 6) from the incus further reduced the stiffness and resistance. From theses results it appears that the tensor tympani and annular ligament have only a small resistive component, and that the major resistive component in the eardrum impedance is due to the cochlea. In summary, it seems likely that for the normal ear, the cochlear loss is largely due to the basilar membrane resistance (since the fluid resistance is believed to be small, based on theoretical estimates).

From these experimental results, it appears feasible that basilar membrane viability might be estimated from the ear canal by human hearing impair subjects. Such a correlation would have important clinical diagnostic applications.

5 References

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